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PHYSICS CALCULATIONS RELATED TO FALLING OBJECTS

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The physical characteristics of various falls and associated calculations related to fall dynamics and force of impact (potential severity) illustrate the critical importance of fall prevention and the use of various fall protection control structures, devices, and activities.

VELOCITY UPON IMPACT (v)

$$v = (v_o^2 + 2gs)^{1/2} \quad \text{or} \quad v = \sqrt{(v_o^2 + 2gs)}$$

where:

- **v** = velocity upon impact (ft/s)
- **v_o** = initial velocity (ft/s)
- **g** = acceleration due to gravity (32.2 ft/s^2)
- **s** = distance of the fall (ft)

RATE OF DECELERATION (a)

$$a = \frac{v^2}{2d}$$

where:

- **a** = rate of deceleration (ft/s^2)
- **v** = the velocity at the point of impact (ft/s)
- **d** = deceleration distance (ft)

G-FORCE (G) (Conversion of rate of deceleration to G-force)

$$G = \frac{a}{g}$$

where:

- **G** = G-force
- **a** = rate of deceleration (ft/s^2)
- **g** = acceleration due to gravity (32.2 ft/s^2)

FORCE OF IMPACT (F_i)

$$F_i = \frac{Wa}{g} = WG$$

where:

- **F_i** = force of impact (pounds force)
- **W** = object weight (lbs)
- **a** = rate of deceleration (ft/s^2)
- **g** = acceleration due to gravity (32.2 ft/s^2)
- **G** = G-force

PHYSICS CALCULATIONS RELATED TO FALLING OBJECTS (CONT'D)

TIME TO FALL (t) BASED ON DISTANCE OF FALL (s)

$$t = \sqrt{\frac{2s}{g}}$$

where:

t = time (sec)
s = distance (ft)
g = 32.2 ft/s^2

PRESSURE OF IMPACT (P_i)

$$P_i = \frac{F_i}{A_i}$$

where:

$\mathbf{P_i}$ = pressure of impact (force per unit area in lbs/in^2)
$\mathbf{F_i}$ = force of impact (pounds force)
$\mathbf{A_i}$ = surface area of impact (in^2)

ABILITY OF OBJECTS TO WITHSTAND FORCE OF IMPACT

$$F = sA$$

where:

s = stress absorption characteristics of the impacted material
A = the area over which the force is applied

Regarding the ability of objects to withstand a force and pressure of impact without injury, one must compare the induced stress of impact (force of impact) to the tensile, compression, shear, puncture, and bending stress that the object being impacted can withstand without damage. Injury to the human body can occur due to direct impact forces, or due to transferred energy to underlying structures when such impact force is transferred to other elements of the body, such as muscles, ligaments, bones, and joints. The time available for the absorption of impact forces (the absorption rate) will also affect the degree of injury.

SEE PAGES 3-8 FOR EXAMPLE CALCULATIONS
SEE PAGE 9 (Appendix A) FOR COMPARISONS TO OTHER PRESSURES
SEE PAGE 10 (Appendix B) FOR INFORMATION ON (IN)^2 VS. SQ. IN.

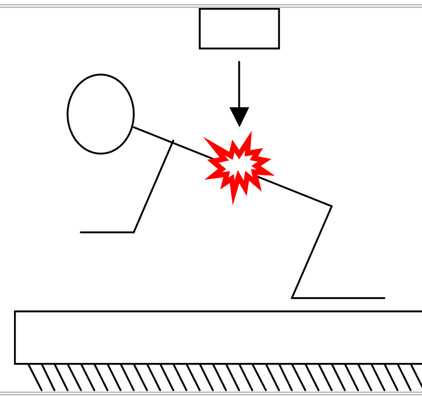
NOTE: If the object weight is greater than indicated, force and pressure of impact will increase.
If the object weight is less than indicated, force and pressure of impact will decrease.
If the fall distance is greater than indicated, force and pressure of impact will increase.
If the fall distance is less than indicated, force and pressure of impact will decrease.
If the deceleration distance is greater than indicated, force and pressure of impact will decrease.
If the deceleration distance is less than indicated, force and pressure of impact will increase.
If the area of impact is greater than indicated, pressure of impact will decrease.
If the area if impact is less than indicated, pressure of impact will increase.

EXAMPLE CALCULATIONS

Example 1

SCENARIO

A 6 pound brick, 6 inches by 4 inches by 2 inches, falls 11 feet, and the 6 inch (right angle) edge strikes a man in the back who is performing work in a kneeling position, assuming 1/4 inch deceleration distance.



VELOCITY UPON IMPACT (v)

$$v = (v_o^2 + 2gs)^{1/2} \quad \text{or} \quad v = \sqrt{(v_0^2 + 2gs)}$$

where:

v = velocity upon impact (ft/s)
$\mathbf{v_o}$ = initial velocity (ft/s)
g = acceleration due to gravity (32.2 ft/s^2)
s = distance of the fall (ft)

$$v = \sqrt{[0 + 2(32.2)(11)]}$$

v = 26.62 ft/s

RATE OF DECELERATION (a)

$$a = \frac{v^2}{2d}$$

where:

a = rate of deceleration (ft/s^2)
v = the velocity at the point of impact (ft/s)
d = deceleration distance (ft)

$$a = \frac{(26.62)^2}{2(1/4 * 1/12)}$$

a = 17,002 ft/s^2

G-FORCE (G) *(Conversion of rate of deceleration to G-force)*

$$G = \frac{a}{g}$$

where:

G = G-force
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)

G = 17002/32.2
G = 528

FORCE OF IMPACT (F_i)

$$F_i = \frac{Wa}{g} = WG$$

where:

$\mathbf{F_i}$ = force of impact (pounds force)
W = object weight (lbs)
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)
G = G-force

F_i = 6 x 528
F_i = 3,168 lbs force

TIME TO FALL (t) BASED ON DISTANCE OF FALL (s)

$$t = \sqrt{\frac{2s}{g}}$$

where:

t = time (sec)
s = distance (ft)
g = 32.2 ft/s^2

$$t = \sqrt{\frac{2(11)}{32.2}}$$

t = 0.83 seconds

PRESSURE OF IMPACT (P)

$$P_i = \frac{F_i}{A_i}$$

where:

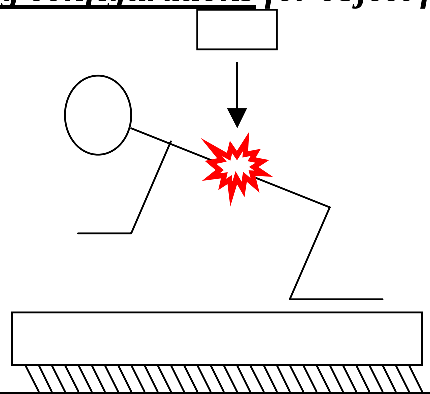
$\mathbf{P_i}$ = pressure of impact (force per unit area in lbs/in^2)
$\mathbf{F_i}$ = force of impact (pounds force)
$\mathbf{A_i}$ = surface area of impact (in^2)

$$P_i = \frac{3168}{1.5}$$

P_i = 2,112 lbs/in^2

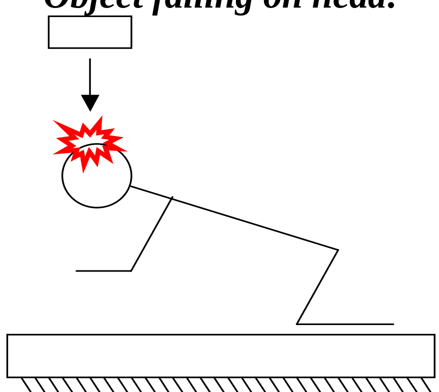
Example 1 (cont.) – VARIATIONS IN THE SCENARIO:

Various __striking configurations__ for object falling on back:



Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
5	17.94	1/4	7,728	6.00	1,440	0.5573	24	60	Strikes flat
5	17.94	1/4	7,728	6.00	1,440	0.5573	1 1/2	960	Strikes on 6" edge
5	17.94	1/4	7,728	6.00	1,440	0.5573	1/16	23,040	Strikes on corner
11	26.62	1/4	17,002	6.00	3,168	0.8266	24	132	Strikes flat
11	26.62	1/4	17,002	6.00	3,168	0.8266	1 1/2	2,112	Strikes on 6" edge
11	26.62	1/4	17,002	6.00	3,168	0.8266	1/16	50,688	Strikes on corner

Object falling on head:



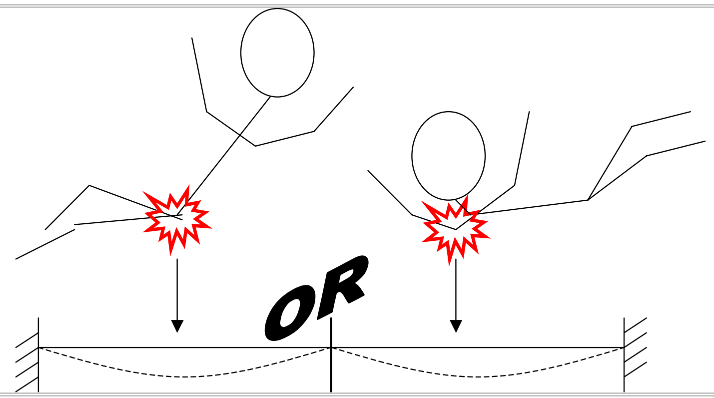
Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
5	17.94	1/4	7,728	6.00	1,440	0.5573	1/8	11,520	Strikes flat
5	17.94	1/4	7,728	6.00	1,440	0.5573	1/8	11,520	Strikes on 6" edge
5	17.94	1/4	7,728	6.00	1,440	0.5573	1/8	11,520	Strikes on corner
11	26.62	1/4	17,002	6.00	3,168	0.8266	1/8	25,344	Strikes flat
11	26.62	1/4	17,002	6.00	3,168	0.8266	1/8	25,344	Strikes on 6" edge
11	26.62	1/4	17,002	6.00	3,168	0.8266	1/8	25,344	Strikes on corner

EXAMPLE CALCULATIONS

Example 2

SCENARIO

A worker weighing 172 pounds falls 11 feet to land on his hip or shoulder on a *<u>surface that deflects 1 foot upon impact</u>*.



VELOCITY UPON IMPACT (v)

$$v = (v_o^2 + 2gs)^{1/2} \quad \text{or} \quad v = \sqrt{(v_0^2 + 2gs)}$$

where:

v = velocity upon impact (ft/s)
v_o = initial velocity (ft/s)
g = acceleration due to gravity (32.2 ft/s^2)
s = distance of the fall (ft)

$$v = \sqrt{[0 + 2(32.2)(11)]}$$

v = 26.62 ft/s

RATE OF DECELERATION (a)

$$a = \frac{v^2}{2d}$$

where:

a = rate of deceleration (ft/s^2)
v = the velocity at the point of impact (ft/s)
d = deceleration distance (ft)

$$a = \frac{(26.62)^2}{2(12 * 1/12)}$$

a = 354 ft/s^2

G-FORCE (G) *(Conversion of rate of deceleration to G-force)*

$$G = \frac{a}{g}$$

where:

G = G-force
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)

G = 354/32.2
G = 11

FORCE OF IMPACT (F_i)

$$F_i = \frac{Wa}{g} = WG$$

where:

F_i = force of impact (pounds force)
W = object weight (lbs)
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)
G = G-force

F_i = 172 x 11
F_i = 1,892 lbs force

TIME TO FALL (t) BASED ON DISTANCE OF FALL (s)

$$t = \sqrt{\frac{2s}{g}}$$

where:

t = time (sec)
s = distance (ft)
g = 32.2 ft/s^2

$$t = \sqrt{\frac{2(11)}{32.2}}$$

t = 0.83 seconds

PRESSURE OF IMPACT (P)

$$P_i = \frac{F_i}{A_i}$$

where:

P_i = pressure of impact (force per unit area in lbs/in^2)
F_i = force of impact (pounds force)
A_i = surface area of impact (in^2)

$$P_i = \frac{1892}{4}$$

P_i = 473 lbs/in^2

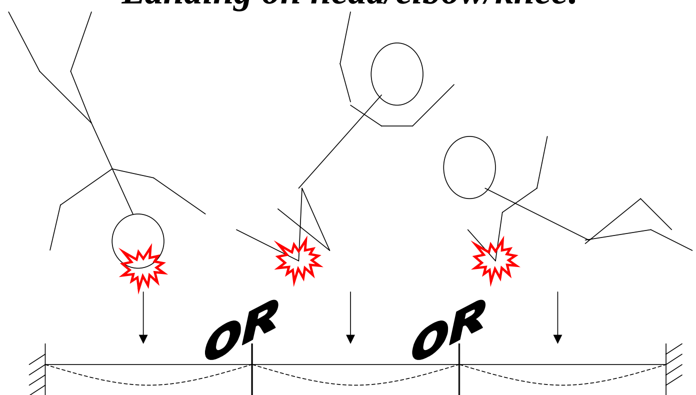
Example 2 (cont.) – VARIATIONS IN THE SCENARIO:

Various worker weights for landing on hip/shoulder on deflecting surface:

Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	12	354	172.00	1,892	0.8266	4	473	Hip/shoulder - male
11	26.62	12	354	137.50	1,513	0.8266	3	504	Hip/shoulder - female

NOTE: Calculations are based upon 50th percentile males and females.

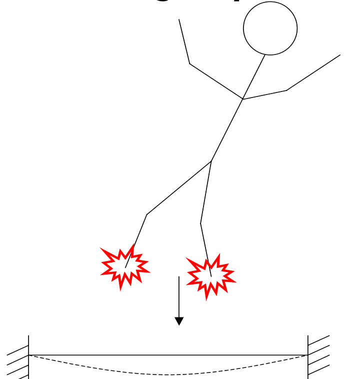
Landing on head/elbow/knee:



Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	12	354	172.00	1,892	0.8266	1/8	15,136	Head/knee/elbow - male
11	26.62	12	354	137.50	1,513	0.8266	1/8	12,100	Head/knee/elbow - female

NOTE: Calculations are based upon 50th percentile males and females.

Landing on feet:



Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	18	236	172.00	1,261	0.8266	16	81	Both feet - male
11	26.62	18	236	137.50	1,008	0.8266	10	104	Both feet - female

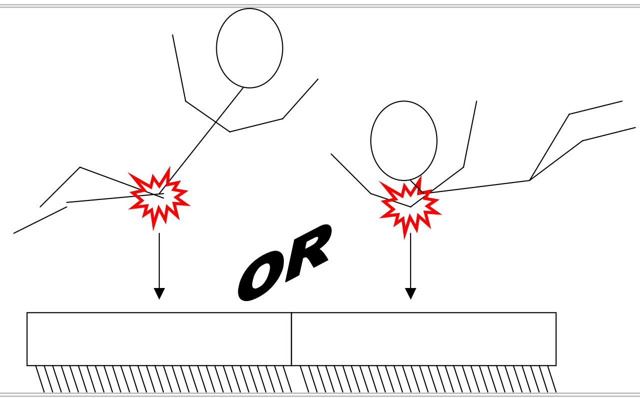
NOTE: Calculations are based upon 50th percentile males and females.
Knee and body flexure of 6 inches during impact is assumed.
It is assumed that the heels absorb the impact, and so are used as the surface area to calculate pressure of impact..

EXAMPLE CALCULATIONS

Example 3

SCENARIO

A worker weighing 172 pounds falls 11 feet to land on his hip or shoulder on a *<u>rigid concrete surface</u>*, assuming a deceleration distance of ½ inch.



VELOCITY UPON IMPACT (v)

$$v = (v_o^2 + 2gs)^{1/2} \text{ or } v = \sqrt{(v_o^2 + 2gs)}$$

where:

v = velocity upon impact (ft/s)
v_o = initial velocity (ft/s)
g = acceleration due to gravity (32.2 ft/s^2)
s = distance of the fall (ft)

$$v = \sqrt{[0 + 2(32.2)(11)]}$$

v = 26.62 ft/s

RATE OF DECELERATION (a)

$$a = \frac{v^2}{2d}$$

where:

a = rate of deceleration (ft/s^2)
v = the velocity at the point of impact (ft/s)
d = deceleration distance (ft)

$$a = \frac{(26.62)^2}{2(1/2 * 1/12)}$$

a = 8,501 ft/s^2

G-FORCE (G) *(Conversion of rate of deceleration to G-force)*

$$G = \frac{a}{g}$$

where:

G = G-force
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)

G = 8501/32.2
G = 264

FORCE OF IMPACT (F_i)

$$F_i = \frac{Wa}{g} = WG$$

where:

F_i = force of impact (pounds force)
W = object weight (lbs)
a = rate of deceleration (ft/s^2)
g = acceleration due to gravity (32.2 ft/s^2)
G = G-force

F_i = 172 x 264
F_i = 45,408 lbs force

TIME TO FALL (t) BASED ON DISTANCE OF FALL (s)

$$t = \sqrt{\frac{2s}{g}}$$

where:

t = time (sec)
s = distance (ft)
g = 32.2 ft/s^2

$$t = \sqrt{\frac{2(11)}{32.2}}$$

t = 0.83 seconds

PRESSURE OF IMPACT (P)

$$P_i = \frac{F_i}{A_i}$$

where:

P_i = pressure of impact (force per unit area in lbs/in^2)
F_i = force of impact (pounds force)
A_i = surface area of impact (in^2)

$$P_i = \frac{45408}{4}$$

P_i = 11,352 lbs/in^2

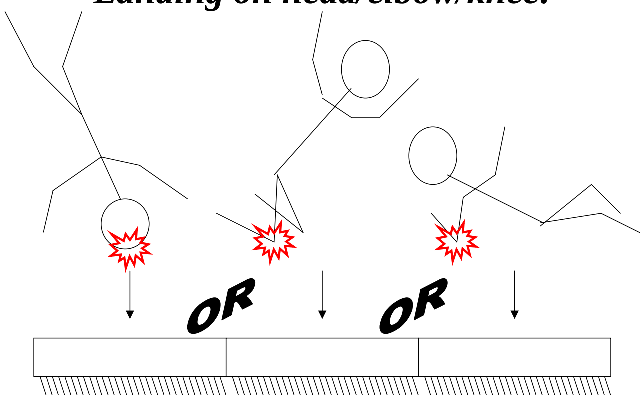
Example 3 (cont.) – VARIATIONS IN THE SCENARIO:

Various worker weights for landing on hip/shoulder on concrete:

Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	1/2	8,501	172.00	45,408	0.8266	4	11,352	Hip/shoulder - male
11	26.62	1/2	8,501	137.50	36,300	0.8266	3	12,100	Hip/shoulder - female

NOTE: Calculations are based upon 50th percentile males and females.

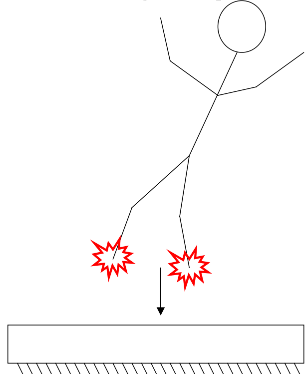
Landing on head/elbow/knee:



Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	1/4	17,002	172.00	90,816	0.8266	1/8	726,528	Head/knee/elbow - male
11	26.62	1/4	17,002	137.50	72,600	0.8266	1/8	580,800	Head/knee/elbow - female

NOTE: Calculations are based upon 50th percentile males and females.

Landing on feet:

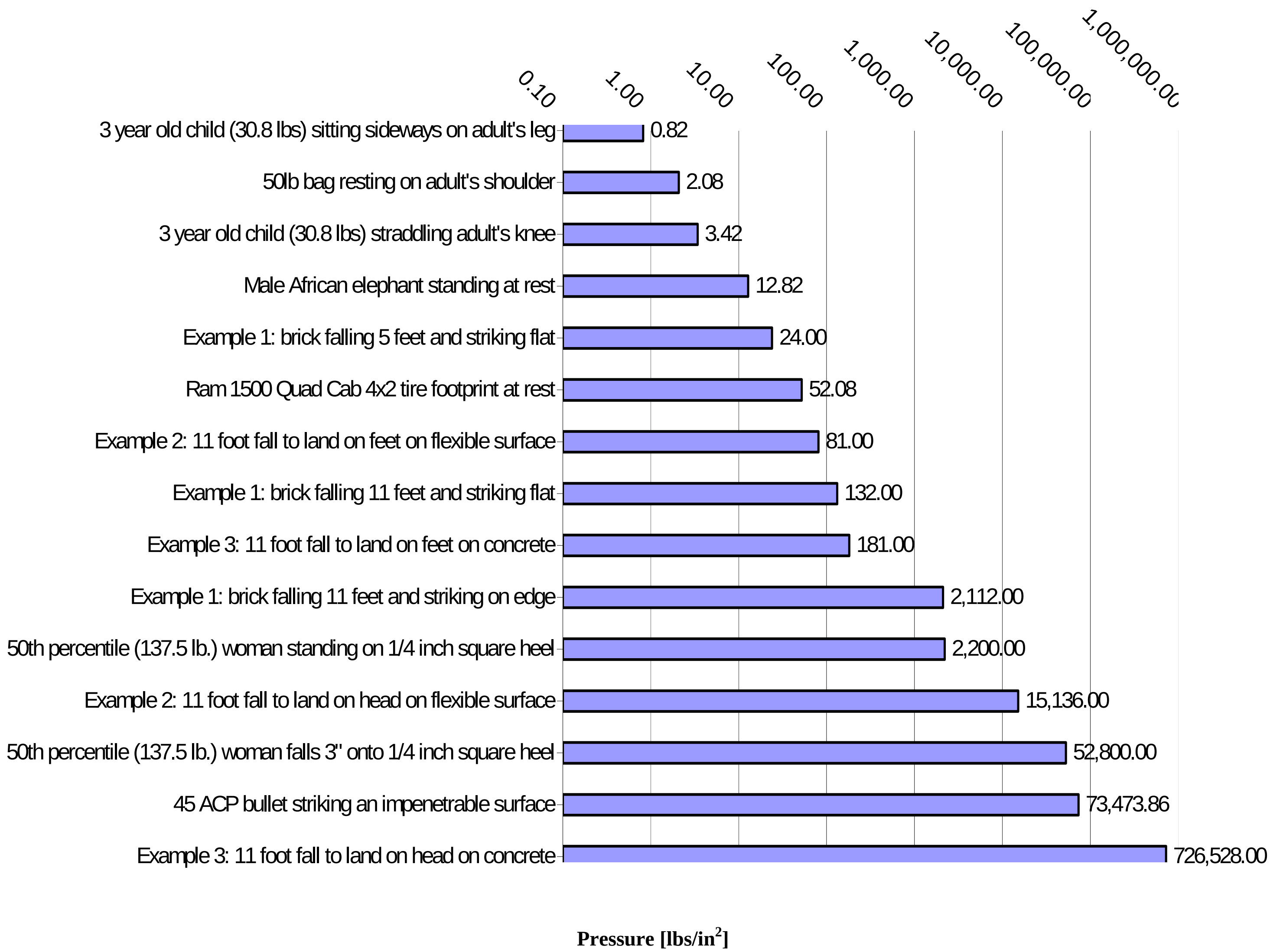


Distance of Fall (ft)	Velocity on Impact (ft/s)	Deceleration Distance (in)	Rate of Deceleration (ft/s^2)	Weight (lbs-force)	Force of Impact (lbs-force)	Time (s)	Area of Impact (in^2)	Pressure of Impact (lbs/in^2)	NOTES
11	26.62	8	531	172.00	2,838	0.8266	16	181	Both feet - male
11	26.62	8	531	137.50	2,269	0.8266	10	234	Both feet - female

NOTE: Calculations are based upon 50th percentile males and females.
Knee and body flexure of 8 inches during impact is assumed.
It is assumed that the heels absorb the impact, and so are used as the surface area to calculate pressure of impact..

APPENDIX A

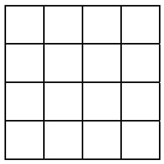
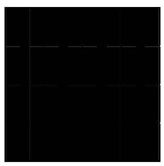
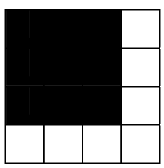
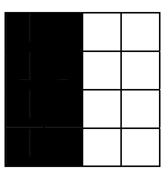
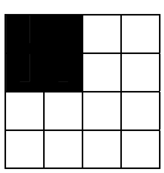
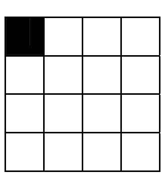
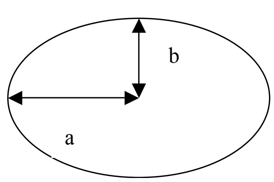
Comparison of pressures in example cases to various other pressures



NOTE: Pressure is displayed in a logarithmic form to allow a wide range of pressures to be shown.
In a logarithmic chart, each mark represents an increase by a magnitude of 10, i.e. 10, 100, 1000, etc.

APPENDIX B

Inches Square (in)^2 vs. Square Inches (in.^{2})

	Inches Square (in)^2	**Square Inches (in.^{2})**
	An <u>area</u> that is <u>also</u> <u>square</u>	Purely an area Does not need to be a square
	= One inch square (1 inch square) = (1 inch)^2 = (1.00 inches)^2	= One square inch (1 sq. in.) = 1.00 square inches (1.00 sq. in.) = (1 in.^{2})
	= Three-quarter inch square = ¾ inch square = (3/4 in)^2 = (0.75 in)^2	= 9/16 square inches = 0.5625 square inches = (0.5625 in.^{2})
	Not a square (in.)^2 not relevant	= One-half square inch = ½ square inch (½ sq. in.) = 0.5 square inches = (1/2 in^2) = (0.5 in^2)
	= One-half inch square = ½ inch square = (1/2 in)^2 = (0.5 in)^2	= One-quarter square inch = ¼ square inch (¼ sq. in.) = 0.25 square inch = (1/4 in.^{2}) = (0.25 in.^{2})
	= One-quarter inch square = ¼ inch square = (1/4 in)^2 = (.25 in)^2	= One-sixteenth square inch = 1/16th square inch (1/16 sq. in.) = 0.0625 square inch = (1/16 in^2) = (0.0625 in^2)
	Not a square (in.)^2 not relevant	Area of ellipse = π a b = 3.1416 x ¾ in. x ½ in. = 3.1416 x .75 in. x .5 in. = 1.18 square inches = (1.18 in^2)

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        - pp. 11- 12: 50th percentile man.
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